Neutrino mass in GUT constrained supersymmetry with R-parity violation in light of neutrino oscillations

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The neutrino masses are generated in grand unified theory (GUT) constrained supersymmetric model with R-parity violation. The neutrinos acquire masses via tree-level neutrino-neutralino mixing as well as via one-loop radiative corrections. The theoretical mass matrix is compared with the phenomenological one, which is reconstructed by using neutrino oscillation and neutrinoless double beta decay data. This procedure allows to obtain significantly stronger constraints on R-parity breaking parameters than those existing in the literature. The implication of normal and inverted neutrino mass hierarchy on the sneutrino expectation values, lepton-Higgs bilinear and trilinear R-parity breaking couplings is also discussed.

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I. INTRODUCTION

The discovery of neutrino oscillations triggered a natural interest in the problem of masses of neutrinos. Unfortunately, in the oscillation experiments only the differences of the squares of masses can be determined. The importance of knowing the absolute scale of neutrino masses is obvious. This knowledge will allow to set direction in which the changes of the standard model of particles and interactions should go; what is more, the problems of dark matter and dark energy, lepto- and baryogenesis, evolution of the Universe and many other could be addressed. Even if the values of neutrino masses will be measured in experiments, the question about the mechanism of obtaining those masses remains open, just in the same way as the widely approved Higgs mechanism is still not experimentally confirmed.

There are many proposals of generating the neutrino mass matrix. Starting from ad-hoc ansätze, through the most-widely approved see-saw model, through extra dimensions, through a result of supersymmetry breaking. Among these one also finds the loop mechanism for Majorana neutrinos. The effective vertex of the form $\bar{\nu}\nu$ is there expanded to contain a squark-quark or slepton-lepton loop. This setting introduces the R-parity violation, therefore it needs to be described within a supersymmetric model with explicitly or spontaneously broken R-parity. Such models provide an elegant way of not only resolving the naturally small neutrino mass problem, but also introduce supersymmetry, needed by the string theory, solve the hierarchy problem, provide much better description of the anomalous magnetic moment of the muon and much more.

The problem of R-parity violation in supersymmetric models received a great deal of attention during last few years [1]. The studies of this topic were connected with, among others, leptonic decays [2], gravitino decays [3] and the problem of neutrino masses and oscillations [4, 5, 6]. In general, supersymmetric models with R-parity violation (RpV) fall into one of the three categories. First, we have the spontaneous RpV. In this case the R-parity is violated by a non-zero vacuum expectation value of some scalar field [7]. Another possibility is the explicit breaking of R-parity by introduction of bilinear and/or trilinear terms. The bilinear RpV models [8] are characterized by good predictivity due to a small number of parameters. The third category is the explicit RpV by trilinear terms present in the superpotential [9, 10]. In this case one allows for the presence of bilinear terms, since these would anyway show up during the RGE evolution of trilinear coupling constants, assuming at the same time that they do not affect the phenomenology of the trilinear terms. This is motivated by the fact that there is no fine tuning among different contributions, which can therefore be analyzed separately. The trilinear scenarios are the most studied due to the reachest phenomenology and possibility of obtaining most interesting limits on non-standard physics parameters.

Our work follows the line of research concerning the generation of neutrino masses in SUSY without R-parity, that has been developed in the last few years. The aim of our paper is to get new individual limits on the R-parity breaking parameters by taking the advantage of the recent data on neutrino oscillations and neutrinoless double beta decay. In addition, the previous studies presented in [6, 10, 11] are improved by a more accurate treatment of the neutrino mass contributions, in particular by reducing the dependence on the SUSY parameter space. The considered model is the minimal supersymmetric

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standard model with supersymmetry breaking transmitted by (super)gravity interactions (SUGRA MSSM) [12], with the squark and slepton mixing phenomena properly included. The RGE evolution within GUT constrained SUGRA MSSM is introduced to obtain the low-energy particle spectrum. The GUT constraints involve unifying masses and coupling constants to some common values at the GUT scale. These are universal scalar mass m_0, gaugino mass $m_{1/2}$, trilinear scalar coupling A_0, the ratio of the Higgs vacuum expectation values $\tan\beta$ and the sign of the bilinear Higgs mixing parameter μ.

The paper is organized as follows. In the next section we present the model and describe our procedure of finding low energy spectrum of SUSY particles using the GUT constraints. We present also improved versions of contributions of different loops to the neutrino mass matrix. In Sec. III we present upper limits on various combinations of RpV coupling constants, both the trilinear λ's and dimensionful bilinears Λ's. Discussion and conclusions follow at the end.

II. THE MODEL

The loop mass mechanism may be described in the framework of R-parity violating MSSM (RpVMSSM) with trilinear and bilinear soft breaking terms. The MSSM (see e.g. [12]) and its many variations are well known in the literature. Here, we closely follow Ref. [13] regarding the conventions and construction of the mass matrices. In short, RpVMSSM is characterized by the superpotential which consists of the R-parity conserving part

$$\begin{aligned} W^{MSSM} &= \epsilon_{ab}[(\mathbf{Y}_E)_{ij} L_i^a H_1^b \bar{E}_j + (\mathbf{Y}_D)_{ij} Q_i^{ax} H_1^b \bar{D}_{jx} \\ &+ (\mathbf{Y}_U)_{ij} Q_i^{ax} H_2^b \bar{U}_{jx} + \mu H_1^a H_2^b], \end{aligned} \tag{1}$$

and the R-parity violating part

$$\begin{aligned} W^{\not R_p} &= \epsilon_{ab}\left[\frac{1}{2}\lambda_{ijk} L_i^a L_j^b \bar{E}_k + \lambda'_{ijk} L_i^a Q_j^{xb} \bar{D}_{kx}\right] \\ &+ \frac{1}{2}\epsilon_{xyz}\lambda''_{ijk} \bar{U}_i^x \bar{D}_j^y \bar{D}_k^z + \epsilon_{ab}\kappa^i L_i^a H_2^b. \end{aligned} \tag{2}$$

The $\mathbf{Y}$'s are 3×3 Yukawa matrices. L and Q are the $SU(2)$ left-handed doublets while $\bar{E}$, $\bar{U}$ and $\bar{D}$ denote the right-handed lepton, up-quark and down-quark $SU(2)$ singlets, respectively. H_1 and H_2 mean two Higgs doublets. We have introduced color indices $x, y, z = 1, 2, 3$, generation indices $i, j, k = 1, 2, 3$ and the SU(2) spinor indices $a, b, c = 1, 2$. In order to get rid of too rapid proton decay and to describe lepton number violating processes, like the neutrinoless double beta decay, it is customary to set $\lambda'' = 0$.

We supply the model with scalar mass term

$$\begin{aligned} \mathcal{L}^{mass} &= \mathbf{m}^2_{H_1} h_1^\dagger h_1 + \mathbf{m}^2_{H_2} h_2^\dagger h_2 + q^\dagger \mathbf{m}^2_Q q + l^\dagger \mathbf{m}^2_L l \\ &+ u \mathbf{m}^2_U u^\dagger + d \mathbf{m}^2_D d^\dagger + e \mathbf{m}^2_E e^\dagger, \end{aligned} \tag{3}$$

soft gauginos mass term

$$\mathcal{L}^{gaug.} = \frac{1}{2}\left(M_1 \tilde{B}^\dagger \tilde{B} + M_2 \tilde{W_i}^\dagger \tilde{W}^i + M_3 \tilde{g_a}^\dagger \tilde{g}^a + h.c.\right), \tag{4}$$

as well as the supergravity mechanism of supersymmetry breaking, by introducing the Lagrangian

$$\begin{aligned} \mathcal{L}^{soft} &= \epsilon_{ab}[(\mathbf{A}_E)_{ij} l_i^a h_1^b \bar{e}_j + (\mathbf{A}_D)_{ij} q_i^{ax} h_1^b \bar{d}_{jx} \\ &+ (\mathbf{A}_U)_{ij} q_i^{ax} h_2^b \bar{u}_{jx} + B\mu h_1^a h_2^b + B_2 \epsilon_i l_i^a h_2^b], \end{aligned} \tag{5}$$

where lowercase letters stand for scalar components of respective chiral superfields, and 3×3 matrices $\mathbf{A}$ as well as $B\mu$ an B_2 are the soft breaking coupling constants.

All the running parameters are obtained by using the renormalization group equations (RGE) [14, 15]. At the beginning, one evolves all gauge and Yukawa couplings for three generations up to the GUT scale $M_{GUT} \sim 10^{16}$ GeV. We use the one-loop standard model RGE [16] below the mass threshold, where SUSY particles start to contribute, and the MSSM RGE [17] above that scale. The contribution of 2-loop diagrams as well as those coming from the R-parity violating couplings has been proven to be irrelevant in discussions such as ours [18]. The SUSY scale is initially set to 1 TeV for all particles and is dynamically modified together with evolution of their masses. At the GUT scale the masses of all scalars and fermions are set to a common value $m_0 = m_{1/2} = m$. We have considered a “small”, chosen to be 150 GeV, and a “large” (1000 GeV) value of m in our analysis. We unify also the soft trilinear couplings according to $\mathbf{A}_i = A_0 \mathbf{Y}_i$, with $A_0 = 500$ GeV. We postpone the discussion of the influence of A_0 on the results to a forthcoming paper. In the next step we construct all the relevant mass matrices (squark, slepton, chargino and neutralino) and perform RGE evolution of all the quantities back to M_Z scale, taking care of the minimization of the tree-level Higgs potential (important for EWSB breaking) and radiative corrections. After iterating this procedure and obtaining stable values of the parameters, we confront the obtained values with restrictions coming from the present theoretical assumptions and phenomenological data. Those constraints involve (1) finite values of Yukawa couplings at the GUT scale; (2) proper treatment of electroweak symmetry breaking; (3) requirement of physically acceptable mass eigenvalues at low energies; (4) FCNC phenomenology.

The first problem is related to the values of $\tan\beta$ and is checked during the RGE procedure. For very small $\tan\beta$ (< 1.8) the top Yukawa coupling may “explode” before reaching the GUT scale. It follows from the fact that $Y_{top}(M_Z) \sim 1/\sin\beta$. Similarly, other couplings Y_b and Y_τ “blow up” before the GUT scale for $\tan\beta > 50$ because they are proportional to $1/\cos\beta$ at electroweak scale. In our analysis we have kept $\tan\beta \approx 20$ leaving the detailed discussion to a forthcoming paper.

Another theoretical constraint is imposed by the EWSB mechanism. In order to obtain a stable minimum of the scalar potential, the following conditions

must hold:

$$
\begin{aligned}
(\mu B)^2 &> (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2), \\
2B\mu &< 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.
\end{aligned} \tag{6}
$$

They are always checked in our procedure during RGE running, and points which do not fulfill these conditions are rejected. Next restriction comes from the requirement of positive eigenvalues of mass matrices squared at the electroweak scale. The last requirement (see, e.g., [15] for details) comes from the strongly experimentally suppressed FCNC processes and provides the most severe constraints.

The so-obtained low energy spectrum is then used in further calculations. Although, as will be seen, only the squarks and sleptons masses enter the formulas, they depend in a complicated way on all other masses and coupling constants through RGE equations [17]. Therefore a complete and careful treatment is necessary.

The neutrino mass matrix consists in our approach of three main parts:

$$
\mathcal{M}_\nu = \mathcal{M}^{tree} + \mathcal{M}^l + \mathcal{M}^q, \tag{7}
$$

which are the tree level value and the contributions coming from lepton-slepton and quark-squark loops, respectively. We note that there are other terms that may be included in Eq. (7), in particular loops contributions with bilinear insertions [5]. For the sake of simplicity, however, we do not consider them by following the popular approach to get individual limits on R-parity breaking parameters. We adopt the conventional hypothesis that different contributions do not significantly compensate each other and for this reason it is possible to extract limits on individual contributions without knowing the others.

Let us first recall the well known results. In the lowest order, the contribution to the mass matrix reads [10]

$$
\begin{aligned}
\mathcal{M}_{ii'}^{tree} &= \Lambda_i \Lambda_{i'}\ g_2^2 \\
&\times \frac{M_1 + M_2 \tan^2\theta_W}{4(\mu M_W^2 (M_1 + M_2 \tan^2\theta_W)\sin 2\beta - M_1 M_2 \mu^2)},
\end{aligned} \tag{8}
$$

where $\Lambda_i = \mu\langle\tilde{\nu}_i\rangle - \langle H_1\rangle\kappa_i$, and $\langle\tilde{\nu}_i\rangle$ are the vacuum expectation values of the sneutrino fields.

Beyond the tree-level, the Majorana neutrino mass matrix may also be generated by considering one loop self-energy diagrams. The particles that propagate inside the loops are either quark and squark or lepton and slepton. Let us start with the squark-quark loop. The relevant Feynman diagrams are shown on Fig. 1. It is important to note, that one may (and should, if one wants to be very accurate) consider not only the trilinear couplings, but also the mass insertions described by the bilinear terms in the superpotential and Lagrangian (see e.g. [8]). Here, however, we take the conventional approach and assume that the phenomenology of the trilinear contribution (which is more interesting from the point of view of obtaining constraints on the RpV coupling constants) remains unaffected by the contribution coming from bilinear terms. Contrary to earlier approaches [10, 11], we take into account the down-squark mixing exactly

$$
\begin{aligned}
\tilde{d}_L &= \tilde{d}_1 \cos\theta + \tilde{d}_2 \sin\theta, \\
\tilde{d}_R &= -\tilde{d}_1 \sin\theta + \tilde{d}_2 \cos\theta.
\end{aligned} \tag{9}
$$

Here L and R label the left- and right-handed squark states in the weak basis, while the 1 and 2 subscripts denote the two mass eigenstates. The mixing angle is defined by

$$
\begin{aligned}
\sin(2\theta^k) &= 2m_{q^k}(A_k + \mu\tan\beta) \\
&\times\ [(m_{\tilde{q}_L^k}^2 - m_{\tilde{q}_R^k}^2 - 0.34 M_Z^2 \cos(2\beta))^2
\end{aligned}
$$

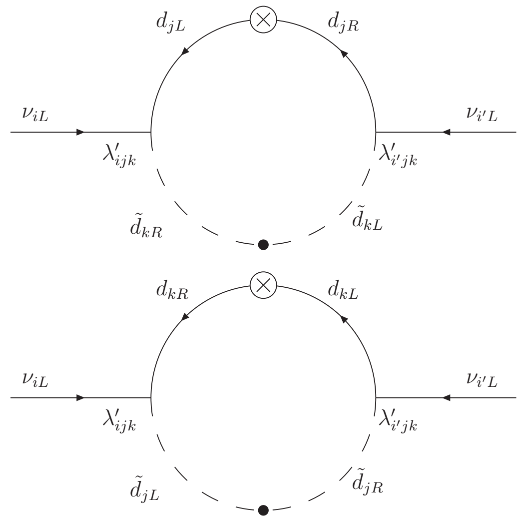

FIG. 1: Feynman diagrams representing the squark-quark loop contribution to the Majorana neutrino mass.

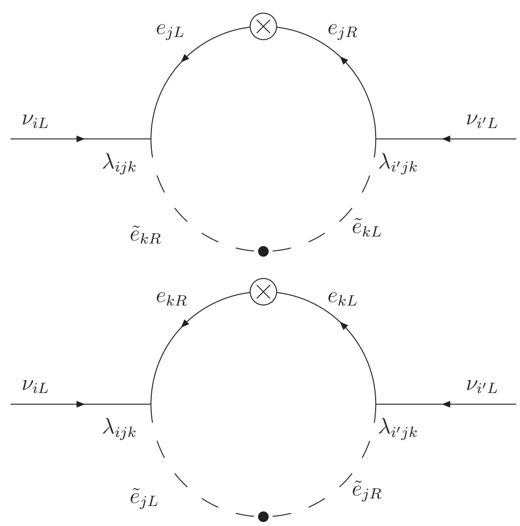

FIG. 2: Feynman diagrams representing the slepton-lepton loop contribution to the Majorana neutrino mass.

$$- \; 4m_{q^k}(A_k + \mu \tan\beta)]^{-1/2} \qquad (10)$$

with $A_k = (\mathbf{A}_D)_{kk}$ and $\tan\beta$ being the ratio of Higgs vacuum expectation values. The squark mass eigenstates take the forms

$$\begin{aligned} m^2_{\tilde{q}^j_1} &= \frac{1}{2}(m^2_{\tilde{q}^j_L} + m^2_{\tilde{q}^j_R}) + m_{q^j}\Big(m_{q^j} - \frac{A_j + \mu\tan\beta}{\sin(2\theta^j)}\Big) \\ &- \frac{1}{4}M_Z\cos(2\beta), \\ m^2_{\tilde{q}^j_2} &= \frac{1}{2}(m^2_{\tilde{q}^j_L} + m^2_{\tilde{q}^j_R}) + m_{q^j}\Big(m_{q^j} + \frac{A_j + \mu\tan\beta}{\sin(2\theta^j)}\Big) \\ &- \frac{1}{4}M_Z\cos(2\beta). \qquad (11) \end{aligned}$$

By introducing two dimensionless quantities, $x_1^{jk} \equiv m^2_{q^j}/m^2_{\tilde{q}^k_1}$ and $x_2^{jk} \equiv m^2_{q^j}/m^2_{\tilde{q}^k_2}$, one arrives at the following form of the neutrino mass matrix

$$\begin{aligned} \mathcal{M}^q_{ii'} &= \frac{3}{16\pi^2}\lambda'_{ijk}\lambda'_{i'kj}\Big[\sin(2\theta^k)m_{q^j} \\ &\times \left(\frac{\log(x_2^{jk})}{x_2^{jk}-1} + \frac{(x_2^{jk}+1)\log(x_1^{jk})}{(x_1^{jk}-1)(x_2^{jk}-1)}\right) \\ &+ (j \leftrightarrow k)\Big]. \qquad (12) \end{aligned}$$

We may repeat the same calculation for the slepton-lepton loop (see Fig. 2), just replacing in all definitions the squark masses and mixing by analogous quantities for sleptons, as well as the quark masses m_{q^j} with lepton masses m_{e^j}. The only difference will be the lack of the factor 3, which came in the previous case from summing over the three colors of quarks and, of course, different coupling constants. We end up with

$$\begin{aligned} \mathcal{M}^l_{ii'} &= \frac{1}{16\pi^2}\lambda_{ijk}\lambda_{i'kj}\Big[\sin(2\phi^k)m_{e^j} \\ &\times \left(\frac{\log(y_2^{jk})}{y_2^{jk}-1} + \frac{(y_2^{jk}+1)\log(y_1^{jk})}{(y_1^{jk}-1)(y_2^{jk}-1)}\right) \\ &+ (j \leftrightarrow k)\Big], \qquad (13) \end{aligned}$$

where now ϕ is the slepton mixing angle, $y_1^{jk} \equiv m^2_{e^j}/m^2_{\tilde{l}^k_1}$ and $y_2^{jk} \equiv m^2_{e^j}/m^2_{\tilde{l}^k_2}$.

Let us now explain the procedure for finding constraints on the various products of coupling constants λ, λ', and Λ. The right hand sides of Eqs. (8), (12), and (13) can be calculated from the MSSM RGE runnigs, during which the low energy particle/sparticle spectrum is generated. We use random scatter to find sets of physically relevant values of the various parameters. In the next step the theoretical neutrino mass matrix is compared with the phenomenological three neutrino mass matrix in the flavor space, which is connected to the physical neutrino masses m_i by the mixing matrix U through the relation $\mathcal{M}^{ph} = U \cdot diag(m_1, m_2, m_3) \cdot U^T$. The standard parameterization of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary matrix U in terms of the three angles is

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}} & 0 \\ 0 & 0 & e^{i\alpha_{31}} \end{pmatrix}, \qquad (14)$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and θ_{ij} is the mixing angle between the flavor eigenstates labeled by indices i and j. The recent global analysis of neutrino oscillations [19] yields the best fit values: $\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 0.5$ and $\sin^2\theta_{13} = 0.002$. Note that for Majorana particles there appear three CP violating phases, one Dirac phase δ and two Majorana phases (α_{21} and α_{31}), which remain undetermined. Assuming the CP phases to be negligible one gets

$$U = \begin{pmatrix} 0.83 & 0.55 & 0.07 \\ -0.42 & 0.55 & 0.72 \\ 0.35 & -0.63 & 0.69 \end{pmatrix}. \qquad (15)$$

The absolute scale of neutrino masses is not determined by the neutrino oscillations, which depend only on differences of masses squared. From the global analysis [19] of neutrino oscillations the best fit values $\Delta m^2_{21} = 6.9\ 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.3\ 10^{-3}$ eV^2 are known. The three possible neutrino mass patterns are frequently considered [20]:
i) *The normal hierarchy (NH) of neutrino masses*, which correspond to the case $m_1 \ll m_2 \ll m_3$. Then we have $m_1 \ll \sqrt{\Delta m^2_{21}}$, $m_2 \simeq \sqrt{\Delta m^2_{21}}$ and $m_3 \simeq \sqrt{\Delta m^2_{31}}$.
ii) *Inverted hierarchy (IH) of neutrino masses.* It is given by the condition $m_3 \ll m_1 < m_2$. In the case for neutrino masses we have $m_3 \ll \sqrt{\Delta m^2_{31}}$ and $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{31}}$.
iii) *Almost degenerate neutrino mass spectrum:* $m_1 \simeq m_2 \simeq m_3$. This case does not exclude the possibility that the lightest neutrino is much larger than $\sqrt{\Delta m^2_{31}}$.

The absolute scale of neutrino masses can be determined by the observation of the end-point part of the electron spectrum of Tritium β-decay, the observation of large-scale structures in the early universe and the detection of the neutrinoless double beta decay ($0\nu\beta\beta$-decay), if neutrinos are Majorana particles. The amplitude of the $0\nu\beta\beta$-decay is proportional to the effective Majorana neutrino mass $m_{\beta\beta} = U^2_{e1}\,m_1 + U^2_{e2}\,m_2 + U^2_{e3}\,m_3$. This process has not been seen experimentally until now and the best results have been achieved in the Heidelberg-Moscow (H-M) experiment ($T^{0\nu}_{1/2} \geq 1.9\times 10^{25}$ y) [21]. (Recently, some authors of the H-M collaboration have claimed the experimental observation of the $0\nu\beta\beta$-decay of ^{76}Ge [22]. But the Moscow participants of the H-M collaboration, performing a separate analysis of the data, found no indication in favor of the evidence of the $0\nu\beta\beta$-decay [23]. The disproof or the confirmation of the claim will come from future experiments.) By assuming the nuclear matrix element of Ref. [24] we end up with

$|m_{\beta\beta}| \leq 0.55$ eV. With this additional input limit we can find the maximal allowed values for the matrix elements $\mathcal{M}^{ph}_{ij}$ of the neutrino mass matrix, which are as follows:

$$|\mathcal{M}^{ph-HM}| = \begin{pmatrix} 0.55 & 0.71 & 0.70 \\ 0.71 & 0.65 & 0.70 \\ 0.70 & 0.70 & 0.76 \end{pmatrix} \ \text{eV}. \quad (16)$$

The elements of this matrix were obtained by assuming the whole allowed mass parameter space of neutrinos and all possible CP-phases of the neutrino mass eigenstates [10]. In the calculation we used the best-fit values of neutrino oscillation parameters given in Ref. [19]. The elements of matrix (16) can be used to test various theoretical approaches and allows one to extract limits on certain fundamental parameters. Of course, one can not expect that by diagonalizing of this matrix a relevant information on the masses of neutrinos is obtained as each element of this matrix is a result of analysis of all possible mixing of three neutrinos allowed by the neutrino oscillations and the $0\nu\beta\beta$-decay data.

Instead of taking into account the current limit on $m_{\beta\beta}$ identified with the element $\mathcal{M}^{ph}_{ee}$ we consider also other scenarios by assuming that the normal or inverted hierarchy of neutrino masses is realized in the nature. Then we get

$$|\mathcal{M}^{ph-NH}| = 10^{-4} \ \text{eV} \\ \times \begin{pmatrix} (22.4-27.2) & (0.64-49.4) & (5.16-52.0) \\ (0.64-49.4) & (223-273) & (210-267) \\ (5.16-52.0) & (210-267) & (196-262) \end{pmatrix},$$

$$|\mathcal{M}^{ph-IH}| = 10^{-2} \ \text{eV} \\ \times \begin{pmatrix} (1.86-4.72) & (0.22-3.11) & (0.26-3.05) \\ (0.22-3.11) & (0.62-2.30) & (0.96-2.37) \\ (0.26-3.05) & (0.96-2.37) & (1.31-2.50) \end{pmatrix}.$$

These neutrino mass matrices were calculated by the assumption that the mass of the lightest neutrino is negligible (see the above definitions of the NH and the IH of neutrino masses).

III. RESULTS AND CONCLUSIONS

By confronting the phenomenological neutrino mass matrix $\mathcal{M}^{ph-HM}$, derived from the analysis of the neutrino data, with the theoretical mass matrix calculated within the R-parity breaking MSSM, it is possible to find constraints on various combinations of the lepton number violating λ, λ' and Λ couplings, which enter Eqs. (8), (12), and (13). If $\mathcal{M}^{ph-NH}$ and $\mathcal{M}^{ph-IH}$ neutrino mass matrices are confronted with the theory, one ends up with predictions for the R-parity violation couplings. By considering the maximal values of these matrices the largest possible values of R-parity breaking parameters are obtained. We note that the predictions for R-parity breaking mechanisms associated with normal or inverted mass hierarchy are deduced by the assumption that one given mechanism dominates at a time. However, this scenario might be excluded by other phenomenology. We have used in our analysis the following quark masses: $m_u = 5$ MeV, $m_d = 9$ MeV, $m_s = 175$ MeV, $m_c = 1.5$ GeV, $m_b = 5$ GeV, $m_t = 174$ GeV.

Table I shows improved upper bounds on various combinations of coupling constants of the λ, λ' and Λ types, to be compared with the limits presented in Refs. [6, 11]. The results were obtained for $A_0 = 500$ GeV, $m_0 = m_{1/2} = 150$ GeV and 1000 GeV and for positive μ. We have found a weak dependence of the quantities under discussion on A_0 SUSY parameter. Besides, we have kept $\tan\beta$ large ($\tan\beta \approx 20$), leaving the discussion of the impact of this parameter on the results to the forthcoming paper. In general, the new bounds related to lower limit on the $T^{0\nu}_{1/2}(^{76}\text{Ge})$ and neutrino oscillation data are at least one order of magnitude stronger than those previously given [11]. It is mostly due to the assumption of the gravity-mediated (SUGRA) supersymmetry breaking and partially due to an improved treatment of the squark and slepton mixing. As expected the values of R-parity violating coupling related to normal hierarchy of neutrino masses are significantly suppressed in comparison with those related to the current lower limit on the $0\nu\beta\beta$-decay half-life [21].

The new bounds are surprisingly close to those published in Ref. [6], although the method used by the authors of these papers relayed on many simplifying assumptions. In particular, it involves setting some of the couplings to zero and assuming all other to be of the same order of magnitude. Also the whole mechanism of RGE running as well as GUT constraints were not used. In general the constraints in [6] were $\lambda_{x33}, \lambda'_{x33} \leq 10^{-8}$ which is fully consistent with our results. The bounds on products of individual coupling constants in Tab. I are either of the same order of magnitude or 1–3 orders of magnitude stronger.

A more optimistic scenario appears for the case of inverted hierarchy. In general the corresponding values of product of λ and λ' coupling are about by factor four less stringent as those associated with the most stringent $0\nu\beta\beta$-decay limit on the half-life. We stress again that these values of the R-parity breaking parameters were determined by the condition that a particular R-parity breaking mechanism dominates at a time. In some cases this might be excluded by the phenomenology of other processes. For example, from the R-parity breaking SUSY mechanism of the $0\nu\beta\beta$-decay one gets the upper limit on the parameter λ'_{111} of the order of 10^{-4} [14, 25], what is significantly less than the value presented in Table I.

In summary, we have used the GUT constrained R-parity violating minimal supersymmetric standard model to describe massive neutrinos. The three family neutrino mass matrix was calculated within framework including the tree-level neutrino-neutralino mixing and the one-loop radiative corrections. Then, the theoretical mass

TABLE I: Constraints on λ, λ' and Λ from their contribution to neutrino masses, using recent global analysis of the neutrino oscillation data [19], the currently best experimental limit on the $0\nu\beta\beta$-decay half-life [21], and matrix element of Ref. [24].

	the $0\nu\beta\beta$-decay limit		normal hierarchy		inverted hierarchy	
$m_0 = m_{1/2} =$	150 GeV	1000 GeV	150 GeV	1000 GeV	150 GeV	1000 GeV
$\|\Lambda_e\|^2$ [GeV^2]	1.7×10^{-2}	5.3	9.0×10^{-5}	2.5×10^{-2}	1.5×10^{-3}	4.6×10^{-1}
$\|\Lambda_\mu\|^2$ [GeV^2]	1.7×10^{-2}	5.3	9.1×10^{-4}	2.7×10^{-2}	7.7×10^{-4}	2.3×10^{-1}
$\|\Lambda_\tau\|^2$ [GeV^2]	1.7×10^{-2}	5.3	8.8×10^{-4}	2.6×10^{-2}	8.4×10^{-4}	2.5×10^{-1}
$\lambda'_{111}\lambda'_{111}$	3.2×10^{-3}	2.4×10^{-2}	1.6×10^{-5}	1.2×10^{-4}	2.7×10^{-4}	2.0×10^{-3}
$\lambda'_{122}\lambda'_{122}$	8.4×10^{-6}	6.2×10^{-5}	4.1×10^{-8}	3.1×10^{-7}	7.2×10^{-7}	5.3×10^{-6}
$\lambda_{122}\lambda_{122}$	1.5×10^{-5}	1.1×10^{-4}	7.2×10^{-8}	5.2×10^{-7}	1.2×10^{-6}	9.0×10^{-6}
$\lambda'_{133}\lambda'_{133}$	8.5×10^{-9}	6.0×10^{-8}	4.2×10^{-11}	3.0×10^{-10}	7.3×10^{-10}	5.1×10^{-9}
$\lambda_{133}\lambda_{133}$	4.3×10^{-8}	3.1×10^{-7}	2.1×10^{-10}	1.5×10^{-9}	3.7×10^{-9}	2.6×10^{-8}
$\lambda'_{132}\lambda'_{123}$	2.6×10^{-7}	1.9×10^{-6}	1.3×10^{-9}	9.5×10^{-9}	2.2×10^{-8}	1.6×10^{-7}
$\lambda_{132}\lambda_{123}$	8.0×10^{-7}	5.8×10^{-6}	3.9×10^{-9}	2.8×10^{-8}	6.8×10^{-8}	4.9×10^{-7}
$\lambda'_{133}\lambda'_{233}$	1.1×10^{-8}	7.7×10^{-8}	7.6×10^{-11}	5.3×10^{-10}	4.8×10^{-10}	3.4×10^{-9}
$\lambda'_{132}\lambda'_{223}$	3.3×10^{-7}	2.5×10^{-6}	2.3×10^{-9}	1.7×10^{-8}	1.5×10^{-8}	1.0×10^{-7}
$\lambda'_{123}\lambda'_{232}$	3.3×10^{-7}	2.5×10^{-6}	2.3×10^{-9}	1.7×10^{-8}	1.5×10^{-8}	1.0×10^{-7}
$\lambda'_{122}\lambda'_{222}$	1.1×10^{-5}	8.1×10^{-5}	7.5×10^{-8}	5.6×10^{-7}	4.7×10^{-7}	3.5×10^{-6}
$\lambda_{133}\lambda_{233}$	5.5×10^{-8}	4.0×10^{-7}	8.5×10^{-10}	2.8×10^{-9}	2.4×10^{-9}	1.7×10^{-8}
$\lambda_{123}\lambda_{232}$	1.0×10^{-6}	7.4×10^{-6}	7.1×10^{-9}	5.1×10^{-8}	4.5×10^{-8}	3.2×10^{-7}
$\lambda'_{233}\lambda'_{233}$	1.0×10^{-8}	7.1×10^{-8}	4.2×10^{-10}	3.0×10^{-9}	3.5×10^{-10}	2.5×10^{-9}
$\lambda'_{232}\lambda'_{223}$	3.1×10^{-7}	2.3×10^{-6}	1.2×10^{-8}	9.5×10^{-8}	1.1×10^{-8}	8.0×10^{-8}
$\lambda'_{222}\lambda'_{222}$	9.9×10^{-6}	7.4×10^{-5}	4.2×10^{-7}	3.1×10^{-6}	3.5×10^{-7}	2.6×10^{-6}
$\lambda_{233}\lambda_{233}$	5.1×10^{-8}	3.7×10^{-7}	2.1×10^{-9}	1.5×10^{-8}	1.8×10^{-9}	1.3×10^{-8}
$\lambda'_{133}\lambda'_{333}$	1.1×10^{-8}	7.6×10^{-8}	8.0×10^{-11}	5.6×10^{-10}	4.7×10^{-10}	3.3×10^{-9}
$\lambda'_{132}\lambda'_{323}$	3.3×10^{-7}	2.4×10^{-6}	2.4×10^{-9}	1.8×10^{-8}	1.4×10^{-8}	1.0×10^{-7}
$\lambda'_{123}\lambda'_{332}$	3.3×10^{-7}	2.4×10^{-6}	2.4×10^{-9}	1.8×10^{-8}	1.4×10^{-8}	1.0×10^{-7}
$\lambda'_{122}\lambda'_{322}$	1.1×10^{-5}	8.0×10^{-5}	7.9×10^{-8}	5.9×10^{-7}	4.6×10^{-7}	3.4×10^{-6}
$\lambda_{132}\lambda_{323}$	1.0×10^{-6}	7.3×10^{-6}	7.5×10^{-9}	5.4×10^{-8}	4.4×10^{-8}	3.2×10^{-7}
$\lambda_{123}\lambda_{322}$	1.8×10^{-5}	1.3×10^{-4}	1.4×10^{-7}	1.0×10^{-6}	8.1×10^{-7}	5.9×10^{-6}
$\lambda'_{233}\lambda'_{333}$	1.1×10^{-8}	7.6×10^{-8}	4.1×10^{-10}	2.9×10^{-9}	3.6×10^{-10}	2.6×10^{-9}
$\lambda'_{232}\lambda'_{323}$	3.3×10^{-7}	2.4×10^{-6}	1.2×10^{-8}	9.3×10^{-8}	1.1×10^{-8}	8.3×10^{-8}
$\lambda'_{223}\lambda'_{332}$	3.3×10^{-7}	2.4×10^{-6}	1.2×10^{-8}	9.3×10^{-8}	1.1×10^{-8}	8.3×10^{-8}
$\lambda'_{222}\lambda'_{322}$	1.1×10^{-5}	8.0×10^{-5}	4.1×10^{-7}	3.0×10^{-6}	3.6×10^{-7}	2.7×10^{-6}
$\lambda_{232}\lambda_{323}$	1.0×10^{-6}	7.3×10^{-6}	3.8×10^{-8}	2.7×10^{-7}	3.4×10^{-8}	2.5×10^{-7}
$\lambda'_{333}\lambda'_{333}$	1.2×10^{-8}	8.3×10^{-8}	4.0×10^{-10}	2.8×10^{-9}	3.9×10^{-10}	2.7×10^{-9}
$\lambda'_{332}\lambda'_{323}$	3.6×10^{-7}	2.6×10^{-6}	1.2×10^{-8}	9.1×10^{-8}	1.2×10^{-8}	8.7×10^{-8}
$\lambda'_{322}\lambda'_{322}$	1.1×10^{-5}	8.6×10^{-5}	4.0×10^{-7}	3.0×10^{-6}	3.8×10^{-7}	2.8×10^{-6}
$\lambda_{322}\lambda_{322}$	2.0×10^{-5}	1.4×10^{-4}	7.0×10^{-7}	5.0×10^{-6}	6.7×10^{-7}	4.8×10^{-6}

matrix was compared with the phenomenological one, obtained by using the most recent global analysis of neutrino oscillations data and the lower limit on the half-life of neutrinoless double beta decay of ^{76}Ge. This procedure allowed to improve the upper limits on certain products of R-parity violating couplings, which are up to one order of magnitude more stringent as those previously published [11]. Further on, we assumed the normal and inverted hierarchy of neutrino masses and calculated the corresponding values of R-parity violating parameters of

the SUSY model under consideration. These values can be used in determining the perspectives of finding signal of R-parity violation in different experiments, in particular at colliders. This issue is, however, beyond the scope of this paper.

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